RTU ROLL NO. ....

ARYA GROUP OF COLLEGES

I MID TERM EXAMINATION 2018-19 (I Sem.)

1FY2-02_Engineering Physics

BRANCH: Common to All

Max Marks:- 80 **Time:- 2 hrs.**

PART A (Attempt All)

Q.1	(a)	In what respects Haidinger's fringes in Michelson interferometer distinct in character from fringes in Newton's rings.	
	(b)	Give Physical interpretation of wave function.	
	(c)	Write short note on X-ray diffraction.	5*2
	(d)	Distinguish between Fresnel and Fraunhofer diffraction?	
	(e)	Explain Rayleigh criterion of resolution?	

PART B (Attempt any Four)

Q.2	(a)	Prove that the diameters of dark rings are proportional to the square root of natural numbers in Newton's ring experiment?	
	(b)	Michelson interferometer experiment is performed with a source which has two wavelengths 4882 A^0 and 4886 A^0. By what distance does the mirror have to be moved between two positions of disappearance of fringes?	
	(c)	Derive Schrodinger's time independent wave equation.	4*10
	(d)	A parallel beam of sodium light is incident normally on a plane transmission grating having 4250 lines/cm and a second order spectral line is observed at an angle 30^0. Calculate wavelength of light.	
	(e)	Obtain an expression for the resolving power of plane diffraction grating	
	(f)	Write down the basic postulates of the wave function?	

PART C (Attempt any Two)

Q.2	(a)	Describe the construction and working of Michelson s Interferometer with diagram. Explain how Michelson Interferometer can be set for getting :- i) Localized fringes ii) Circular fringes.	
	(b)	The intensity of light diffracted from a plane transmission grating is given by $I = I_o\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ Where symbols have their usual meanings. Find the positions of maxima and minima.	2*15
	(c)	Discuss the phenomenon of Fraunhofer diffraction at single slit and show that the relative intensities of successive maxima are nearly: $1:\frac{4}{9\pi^2}:\frac{4}{25\pi^2}:\frac{4}{49\pi^2}......$	

(PART-A)

Q. 1.

(a) (I) Haidinger's fringes in M.I. are formed by thin film which is produced b/w the real mirror M_1 and virtual mirror M_2' while the circular fringes in Newton's ring are formed also due to thin film which is b/w the plano-convex lens and plane glass plate but thickness of the thin film is gradually increase from the centre of contact to extreme point of the surface of plano-convex lens and plane glass plate while in M.I the thickness of the thin film is the same.

(II) The effective path difference of Haidinger's fringes in M.I is

$$2d\cos\theta \pm \lambda/2$$

where d is the thickness or seperation b/w the mirror M_1 and virtual mirror M_2' and θ is the angle b/w two virtual source S_1 and S_2 which are due to mirror M_1 and M_2'.

while the effective path difference of Newton's ring is -

$$2\mu t\cos r \pm \lambda/2$$

where

$\mu \rightarrow$ refractive index

$t \rightarrow$ thickness of thin film

$r \rightarrow$ refraction angle

(iii) Haidinger's fringes in M.I. due to equal of inclination while the Newton's ring is due to equal of thickness.

(iv). The centre of Haidinger's fringes are in alternate black/white while in Newton's ring, the centre of circular ring is always dark.

(b). Physical ~~a~~ interpretation of wave function → The wave function is defined by $\psi(x, y, z, t)$ which is function of position and time and it is explained ~~by~~ the behavior of particle at the position x, y, z at the time t. It is complex quantity that's why it does not has any physical significance but it define the probability of the finding particle.

(I). Probability of the wave function → The wave function is a complex quantity and it can be written as.

$$\psi(x, t) = A \exp\left[-\frac{i}{\hbar}(Et - p_x x)\right]$$

Now if

$$\psi(x, y, z, t) = A + iB$$

$$\psi^*(x, y, z, t) = A - iB$$

$$P = \int_{-\infty}^{\infty} |\psi(x, y, z, t) \cdot \psi^*(x, y, z, t)| \, dv = A^2 + B^2$$

it is defined the probability of finding particle

(ii) Normalization of wave function → It is defined as –

$$\int_{-\infty}^{\infty} \psi(r,t)\, \psi^*(r,t)\, dr = 1$$

The probability is unity.

(iii) Orthogonal wave function → It is defined as –

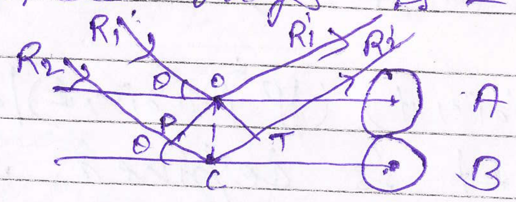
$$\int_{-\infty}^{\infty} \psi_m(r,t)\, \psi_n^*(r,t)\, dr = 0$$

It means the two particles are the perpendicular to each other.

(c) XRD → When x-ray is incident on a crystal then the x-ray will be diffracted from the plane of the atoms in the crystal because of the wavelength of x-ray and the seperation b/w the atoms are same. It is ~~called~~ called the XRD.
If the x-ray is incident on crystal at an angle θ to ~~from~~ the plane A and B then the total path difference b/w reflected rays is –

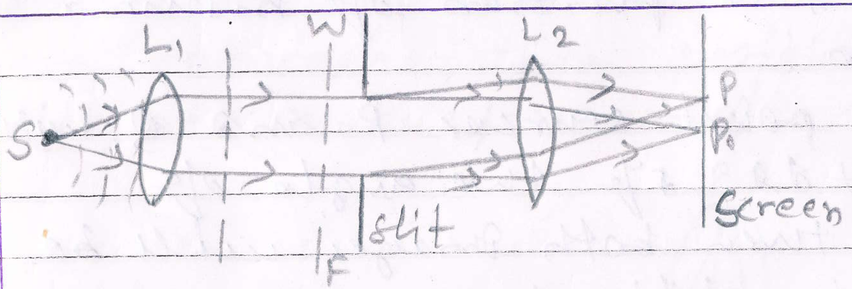
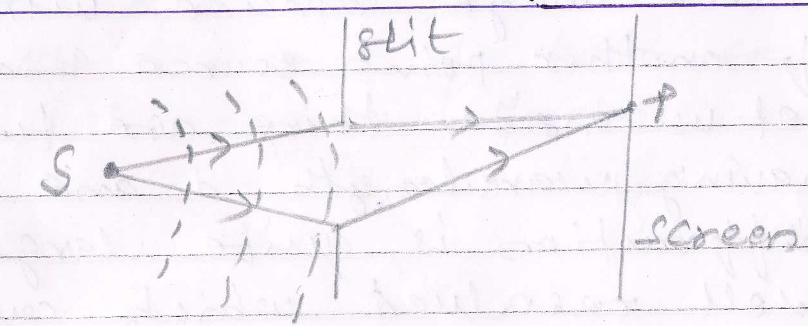
~~$2d$~~

$$\boxed{2d \sin\theta = n\lambda}$$



It is called the bragg's eqn-

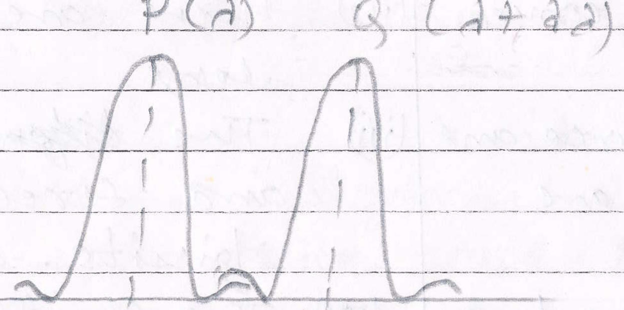
(d)-

Fraunhofer diffraction	Fresnel diffraction
(i) 	(i) 
(ii) There are need two convex lenses	(ii) there are no need any type of lens
(iii) The distance of the source and screen from the slit are infinite	(iii) The ~~distan~~ distance of the source and screen from the slit is finite.
(iv) The incident wavefront on the slit is plane wavefront	(iv) The incident wavefront on the slit is spherical or cylindrical wavefront.
(v) The centre of this pattern is always bright	(v) The centre of this pattern may or not bright.

(e). <u>Rayleigh criterion of resolution</u> ⇒

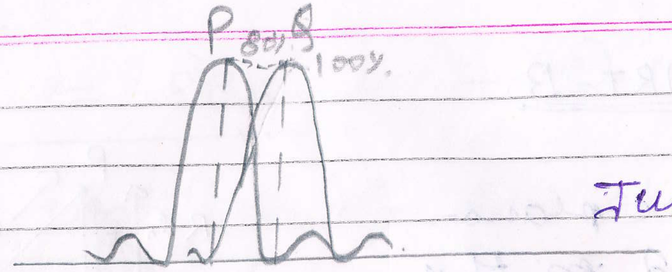
According to the lord Rayleigh, the two point source images are said to be just resolve, if the position of maxima of one point source image coincide with the position of minima of another point source image.

Let us consider there are two point sources P and Q with having wavelength λ and $\lambda + d\lambda$. If the angle of diffraction is quite large, then both images will be well resolved which can be defined as –



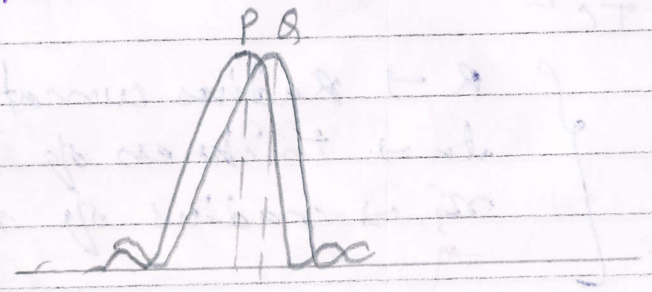
Well resolved

Now, if the difference b/w wavelength of two point source images is decreased and according to Rayleigh criterion, if the position of maxima of P point source coincide with the position of minima of another point source then it will be just resolve.



Just ~~[illegible]~~ resolved

Now, if the difference wavelength b/w two point source is more decreased, then position of maxima of both sources will be overlap and it will be not resolved.



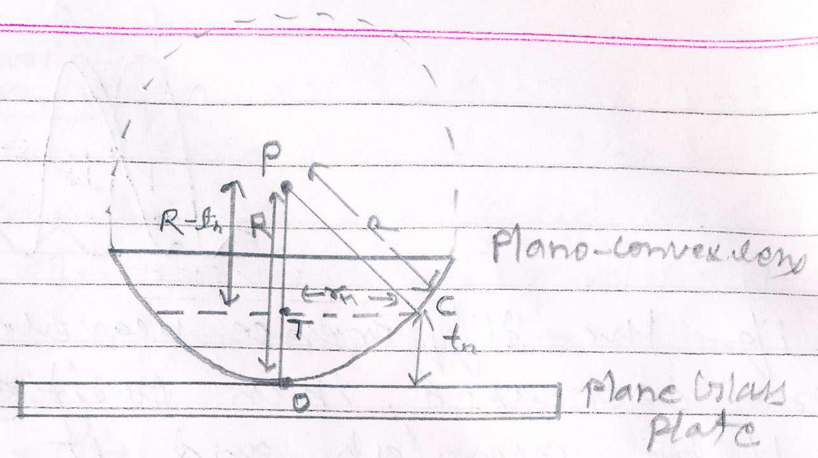
Not resolved

P.T.O

Q. (2).

PART - B

(a). In Newton's Ring, the plano-convex lens is placed on the plane glass plate.



So, From fig

ΔPTC

$$PC^2 = PT^2 + TC^2$$

Here

$PC = R$

$PT = R - t_n$

$TC = r_n$

$\left\{ \begin{array}{l} R \to \text{Radius curvature of lens} \\ t_n \to \text{thickness of thin film of } n^{th} \\ r_n \to \text{radius of } n^{th} \text{ ring.} \end{array} \right.$

$$R^2 = (R - t_n)^2 + r_n^2$$

$$\cancel{R^2} = \cancel{R^2} + t_n^2 - 2R.t_n + r_n^2$$

If $R >> t_n$, then, t_n^2 can be ignored.

So,

$$r_n^2 = 2R.t_n$$

And Diameter

$$D_n = 2r_n \Rightarrow r_n = \frac{D_n}{2} \Rightarrow r_n^2 = \frac{D_n^2}{4}$$

so,

$$\frac{D_n^2}{4} = 2R t_n$$

$$\boxed{D_n^2 = 8R\,t_n} \quad \text{— (1)}$$

As we know the condition of dark ring in Newton's ring -

$$2\mu t_n = n\lambda \qquad \text{for air } \mu = 1$$

$$2t_n = n\lambda$$

$$t_n = n\lambda/2 \quad \text{— (2)}$$

putting in eqⁿ (1)
we have

$$D_n^2 = \cancel{8}^{4}R \times \frac{n\lambda}{\cancel{2}}$$

$$D_n^2 = 4Rn\lambda$$

$$D_n = \sqrt{4Rn\lambda}$$

or $\boxed{D_n \propto \sqrt{n}}$

Proved

The diameter of dark ring is proportional to the square root of natural numbers.

(b) Given

$$\lambda_1 = 4882\ A^\circ$$

$$\lambda_2 = 4886\ A^\circ$$

As we know the formula to determine the difference in wavelength is given by –

$$\lambda_2 - \lambda_1 = \frac{\lambda_{avg}^2}{2d}$$

$$\lambda_{avg} = \frac{4882 + 4886}{2} = 4884\,A^\circ$$

$$d = \frac{\lambda_{avg}^2}{2(\lambda_2 - \lambda_1)}$$

$$\lambda_2 - \lambda_1 = 4886 - 4882 = 4\,A^\circ$$

$$= \frac{(4884)^2 \times 10^{-10} \times 10^{-10}}{2 \times 4 \times 10^{-10}}$$

$$d = \frac{23853456 \times 10^{-10}}{4 \times 2}$$

$$d = \frac{0.596\,mm}{2}$$

$$d = 0.298\,mm$$

(c) Schrondinger's time independent wave eq^n ⇒ As we know the wave function is defined as –

$$\psi(x,t) = A \exp\left[-\frac{i}{\hbar}(Et - P_x x)\right]$$

$$\psi(x,t) = A\, e^{-\frac{i}{\hbar}Et}\, e^{\frac{i}{\hbar}P_x x}$$

$$\psi(x,t) = \psi(x)\, e^{-\frac{i}{\hbar}Et} \quad \text{--- (1)}$$

Here
$$\psi(x) = A\, \exp\left[\frac{i}{\hbar} P_x x\right]$$

∴ differentiate w.r. to x of eq^n (1) we have

$$\frac{\partial \psi(x,t)}{\partial x} = \frac{\partial \psi(x)}{\partial x}\, e^{-\frac{i}{\hbar}Et}$$

once again diff. w.r. to x we have -

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \frac{\partial^2 \psi(x)}{\partial x^2}\, e^{-\frac{i}{\hbar}Et} \quad \text{--- (2)}$$

The Schrondinger's time dependent wave eq^n is given by

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x,t)\,\psi(x,t) \quad \text{--- (3)}$$

Here the potential energy is

$$V(x,t) = V(x) \quad \text{--- (4)}$$

energy operator

$$i\hbar\, \partial/\partial t \rightarrow E \quad \text{--- (5)}$$

Using the eq^n (4), (5) in eq^n (3)
we have

$$E\,\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\,\psi(x,t) \quad \text{--- (6)}$$

Putting the eq^n ①, ② in eq^n ① we have –

$$E\,\psi(x)\,e^{-\frac{i}{\hbar}Et} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}e^{-\frac{i}{\hbar}Et} + V(x)\,\psi(x)\,e^{-i/\hbar\,Et}$$

or

$$E\,\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\,\psi(x)$$

$$\boxed{\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0}$$

It is called the schrodinger's time independent wave eq in 1-D.

(d) Given

$$(e+b) = \frac{1}{4250 \text{ lines}}$$

$$\theta = 30^\circ$$

$$n = 2 \quad ; \quad \lambda = ?$$

As we know the grating eq^n –

$$(e+b)\sin\theta = n\lambda$$

$$\lambda = \frac{(e+b)\sin\theta}{n} \qquad \left\{ \sin 30^\circ = \frac{1}{2} \right\}$$

$$= \frac{1 \times 10^{-2}}{4250 \times 2 \times 2}$$

$$\lambda = 0.05882 \times 10^{-5}\ m$$

$$\lambda = 5882 \times 10^{-10}\ m$$

or $\boxed{\lambda = 5882\ A^\circ}$

(e) Resolving power of plane diffraction grating →

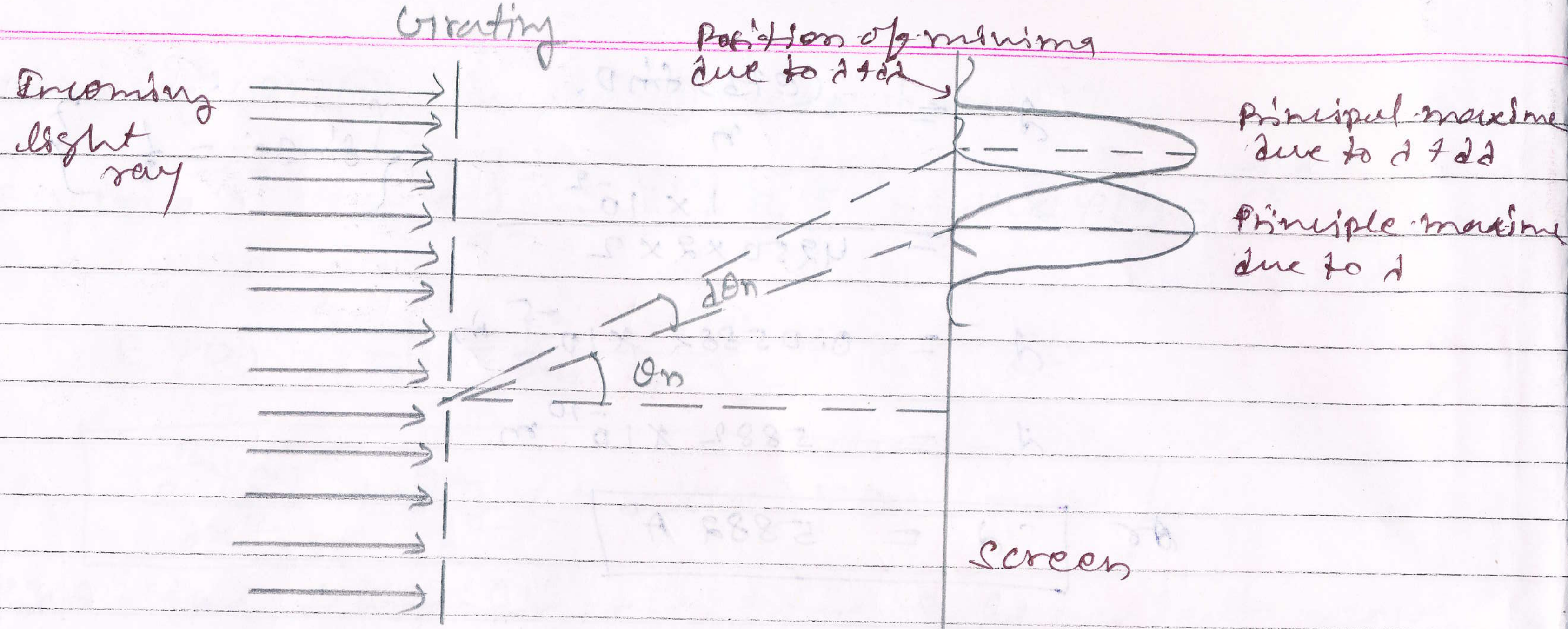
It is defined as the ability to separate the two closed spectral lines or it can be defined as $\lambda / d\lambda$

Let us consider there are two principal maxima having wavelength λ and $\lambda + d\lambda$, when the incoming light ray incident on the grating and diffracted at an angle θ_n.

So, as we know the position of principal maxima is given by

$$(e+b)\sin\theta = n\lambda$$

Now according the Fig -



The position of principal maxima due to $\lambda + d\lambda$ is given by

$$(e+b)\sin(\theta_n + d\theta_n) = n(\lambda + d\lambda) \quad \text{——— (1)}$$

The position of ~~principal~~ minima due to λ is

$$(e+b)\sin(\theta_n + d\theta_n) = \frac{m}{N}\lambda$$

Here for minima

$$m = nN+1$$

so

$$(e+b)\sin(\theta_n + d\theta_n) = \frac{nN+1}{N}\lambda \quad \text{——— (2)}$$

According to the rayleigh criterion the spectral lines of grating is said to be just resolve if the position of principal maxima of one spectral line coincide with ~~the~~ the position of minima of another spectral line. so from eq^n ①, ② we have –

$$n(\lambda + d\lambda) = \frac{nN+1}{N}\lambda$$

$$\cancel{n\lambda} + d\lambda\, n = \cancel{n\lambda} + \frac{\lambda}{N}$$

or $$\frac{\lambda}{d\lambda} = nN$$

or $$\boxed{R = nN.}$$ ~~R~~ R is the resolving power of the grating.

(f) Basic postulates of the wave function ⇒

There are three basic postulates –

(1) Boundary condition →

(A) The wave function $\psi(x,t)$ should be finite value

(B) The wave function $\psi(x,t)$ should be single valued function.

(C) The wave function ψ and its derivate $\frac{d\psi}{dx}, \frac{d\psi}{dy}, \frac{d\psi}{dz}$ should be finite, continuous and differentiate.

② **Eigen value and eigen function** →

The eigen values and the eigen function can be defined as –

$$\hat{A}\psi = a\psi$$

Here $\hat{A}$ is the eigen operator

ψ is the eigen function

a is the eigen value

For example if eigen operator is d/dx and eigen function is e^{8x} then

$$\frac{d}{dx}(e^{8x}) = 8e^{8x}$$

Here 8 is the eigen value.

③ **Expectation value** ⇒

The expectation value is defined as –

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} \psi(x,t)\, x\, \psi^*(x,t)\, dx}{\int_{-\infty}^{\infty} \psi(x,t)\cdot\psi^*(x,t)\, dx}$$

As we know the normalisation condition is –

$$\int_{-\infty}^{\infty} \psi(x,t) \cdot \psi^*(x,t)\, dx = 1$$

By using this eq^n the expectation value will be –

$$\boxed{\langle x \rangle = \int_{-\infty}^{\infty} \psi(x,t)\, x\, \psi^*(x,t)\, dx}$$

PART – C

Q.2.

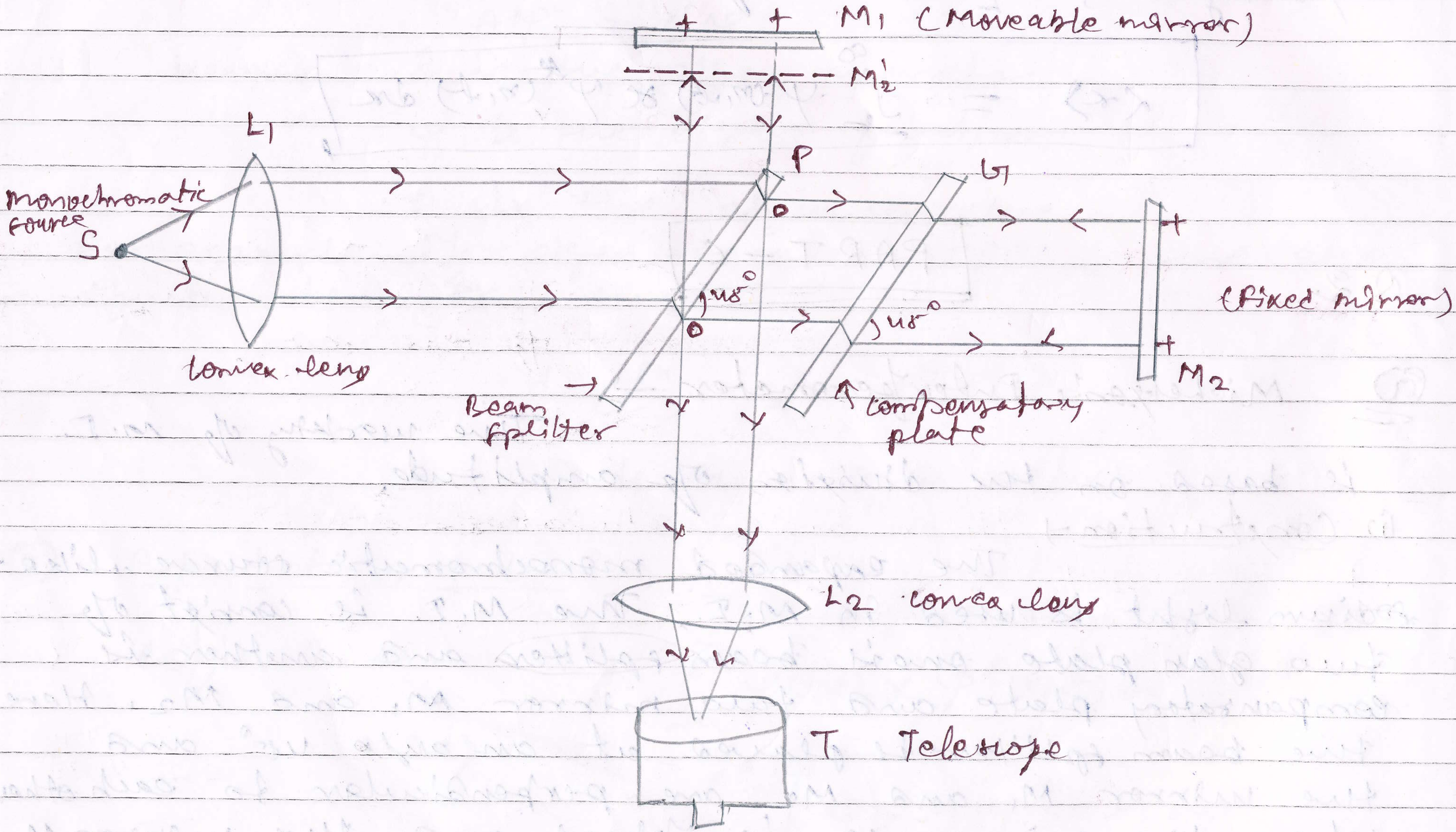
(a) Michelson's Interferometer →

The working of M.I. is based on the division of amplitude.

(i) Construction →

The expanded monochromatic source like sodium light is used in M.I. The M.I. is consist of two glass plate one is beam splitter and another is compensatory plate and two mirror M_1 and M_2. Here the beam splitter is placed at an angle 45° and the mirror M_1 and M_2 are perpendicular to each other where the mirror M_2 is fixed and the mirror M_1

is moveable, it means it can be moved by its screw.
"The incoming light rays reflect back from the mirror M_1 and mirror M_2 and reach at the telescope.

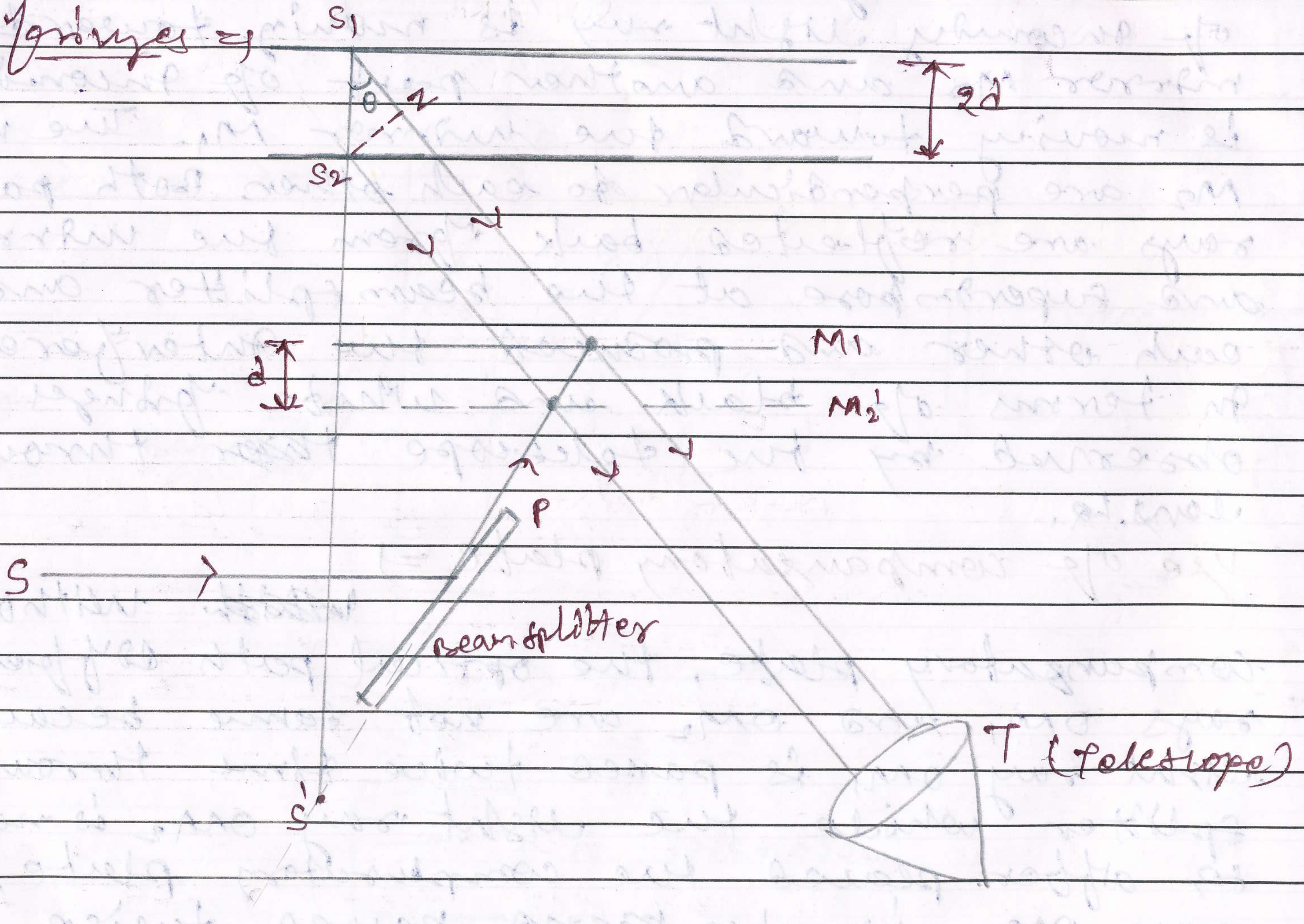


(ii) Working ⇒ When the monochromatic light is incident on the beam splitter through convex lens L_1. The beam splitter is placed at an angle 45° and it is divides the incoming light ray into two parts. The one part of incoming light ray is moving toward to the ~~mirror~~ mirror M_2 and another part of incoming light ray is moving toward the mirror M_1. The mirror M_1 and M_2 are perpendicular to each other. Both parts of light rays are reflected back from the mirror M_1 and M_2 and superimpose at the beam splitter and interfere to each other and produced the interference pattern in terms of black and white fringes which is observed by the telescope ~~tho~~ through the convex lens L_2.

Use of compensatory plate ⇒

~~which~~ Without the compensatory plate, the optical path difference b/w the rays OM_1 and OM_2 are not same because of the light ray OM_1 is passes twice time through the beam splitter while the light ray OM_2 is not passed once. So, after placed the compensatory plate, the light ray OM_2 is also ~~passes~~ passes twice time through the glass plate and produces the equal optical path difference b/w the light ray OM_1 and OM_2. that's why the compensatory plate must be as same thickness, refractive index, width as the beam splitter.

(I) Circular fringes :-



When the mirror M_1 and virtual mirror M_2' are exactly parallel to each other, then the circular fringes are observed. Here if the seperation b/w the mirror M_1 and M_2' is d then the distance b/w S_1 and S_2 is $2d$, where S_1 is the virtual source of the mirror M_1 and S_2 is the virtual source of the virtual mirror M_2'. So, from fig, it is cleared that the extra path length S_1Z is due virtual sources so, from fig:→

ΔS_1ZS_2 →

$$\cos\theta = \frac{S_1Z}{S_1S_2} \quad \left\{ \begin{array}{l} \text{Here} \\ S_1S_2 = 2d \end{array} \right.$$

$$S_1Z = S_1S_2 \cos\theta$$

$$S_1Z = 2d\cos\theta$$

or $$\Delta = 2d\cos\theta \quad \text{—(1)}$$

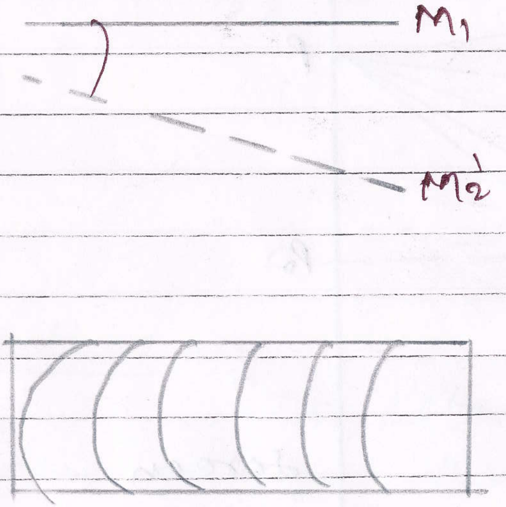
From stokes law the total effective path difference is.

$$\boxed{\Delta_{eff} = 2d\cos\theta \pm \lambda/2}$$

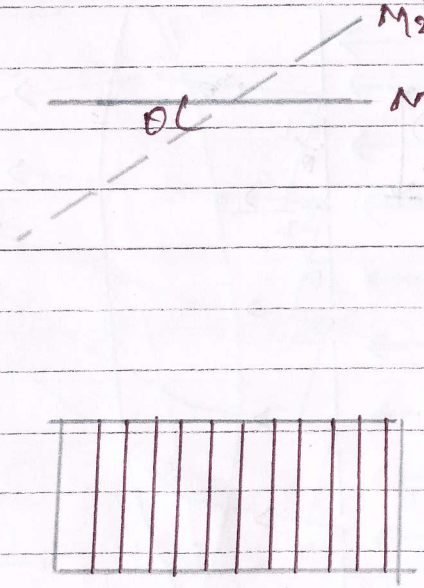
II

(II) Localized fringes ⇒

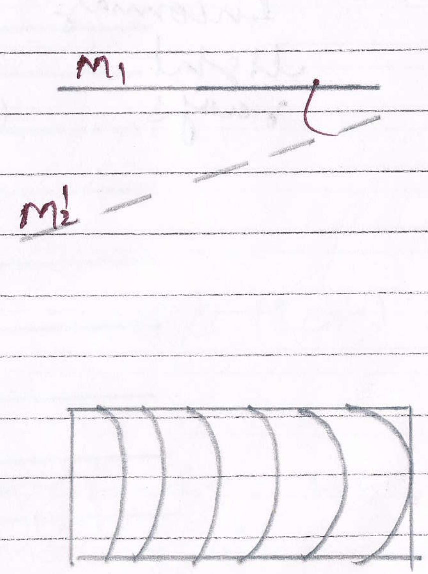
When the mirror M_1 intersect to the virtual mirror M_2' then the straight fringes, parabolic or hyperbolic fringes will be formed. If the mirror M_1 is exactly intersect to M_2' then the straight fringes will be formed and if the mirror M_1 is not exactly intersect to M_2' while inclined at some angle then the parabolic fringes will be formed.



parabolic fringes



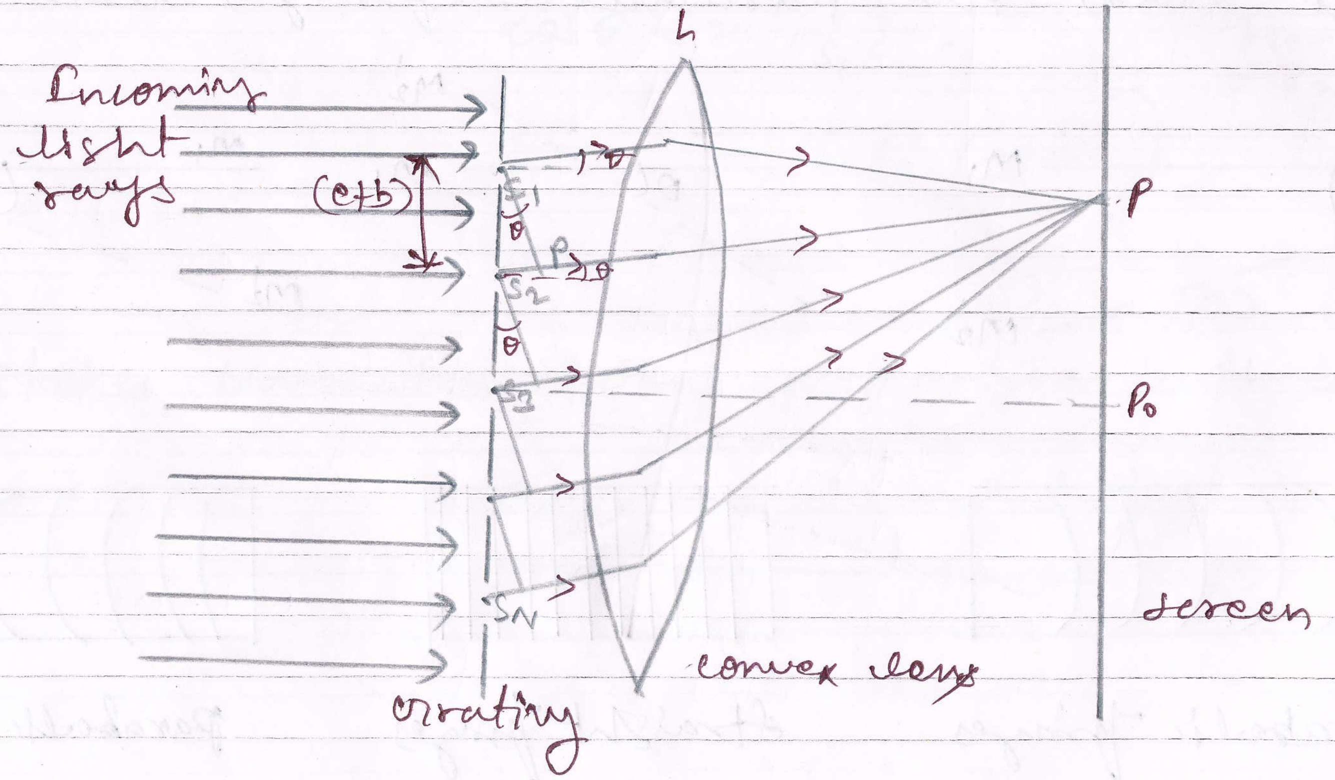
straight fringes



parabolic fringe

(b) Plane transmission grating ⇒

The plane transmission grating is consist of large no. of parallel lines of equal width and equal seperation b/w lines. Now, let us consider the wavelength of λ is incident on the grating then it will be diffracted at angle θ and produced the intensity pattern on the screen.



In grating according to Huygen's principal, there are N sources which are called the secondary sources like $S_1, S_2, S_3 \cdots S_N$. So, from the fig, the total path difference b/w the secondary ~~sources~~ wavelets –

$$\sin\theta = \frac{S_2P}{S_1S_2}$$

$$S_2P = S_1S_2 \sin\theta \quad \left\{ \begin{array}{l} \text{Here} \\ S_1S_2 = e+b \end{array} \right\}$$

$$\text{or} \quad \Delta = (e+b)\sin\theta \; ; \; (e+b) \rightarrow \text{Grating element}$$

The phase difference will be –

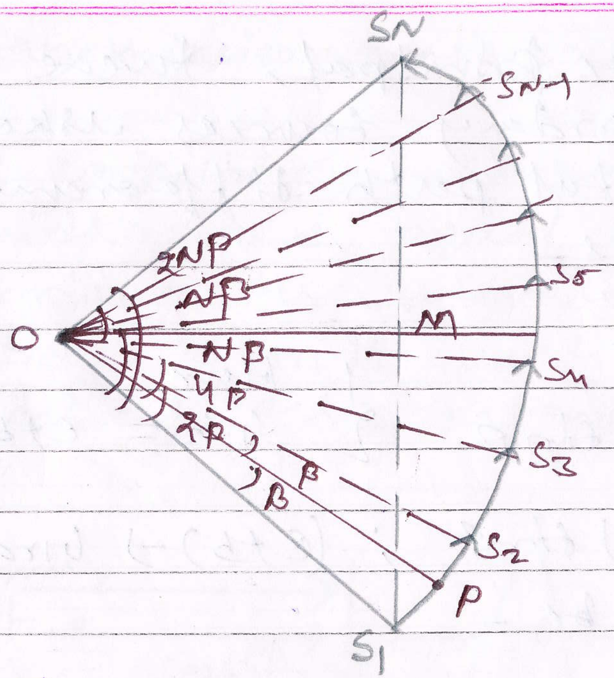
$$\delta = \frac{2\pi}{\lambda} \times \Delta$$

$$\delta = \frac{2\pi}{\lambda} \times (e+b)\sin\theta = 2\beta \text{ (Let)} \quad \text{—— (1)}$$

Polygon Method ⇒

To determine the total amplitude, we have to used the polygon method. According to this $S_1, S_2 \cdots S_N$ are the secondary source and produce the secondary wavelets the set of secondary sources like $S_1S_2, S_2S_3 \cdots S_{N-1}S_N$ act like the single slit which has the amplitude

$$S_1S_2 = S_{N-1}S_N = R_0 \frac{\sin\alpha}{\alpha} \quad \text{where } \alpha = \frac{\pi}{\lambda} e\sin\theta$$



Here

$S_1S_N = 2S_1M = 2S_NM$

$\angle S_NOS_1 = 2N\beta$

$\angle MOS_1 = N\beta$

$S_1S_2 = 2S_1P = 2S_2P$

$\angle POS_1 = \beta$

From the Fig →

ΔOMS_1 →

$$\sin N\beta = \frac{MS_1}{OS_1}$$

$$MS_1 = OS_1 \sin N\beta$$

or $$S_1S_N = 2\,OS_1 \sin N\beta \quad \text{——— (2)}$$

From OPS_1

$$\sin\beta = \frac{S_1P}{OS_1}$$

$$S_1P = OS_1 \sin\beta$$

or $$S_1S_2 = 2\,OS_1 \sin\beta \quad \text{——— (3)}$$

Dividing the eq (2) by eq (3)

$$\frac{S_1S_N}{S_1S_2} = \frac{\cancel{2OS_1}\sin N\beta}{\cancel{2OS_1}\sin\beta}$$

$$S_1S_N = S_1S_2 \frac{\sin N\beta}{\sin\beta} \quad \left\{ \text{Here } S_1S_2 = ka\frac{\sin\alpha}{\alpha} \right\}$$

$$S_1S_N = ka\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin N\beta}{\sin\beta}\right)$$

As we know

$$I \propto R^2 \text{ or } (S_1S_N)^2$$

$$I = k^2a^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

or

$$\boxed{I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2} \quad \text{—(4)}$$

(I) position of maxima →

If $\sin\beta = 0$

$\Rightarrow \beta = \pm n\pi$

From L'Hospital rule –

$$\lim_{\beta\to\pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)} = \lim_{\beta\to\pm n\pi} \frac{N\cos N\beta}{\cos\beta}$$

So, from eqⁿ (4)

$$\boxed{I = I_0 N^2}$$

Here

$$\lim_{\alpha \to 0}\left(\frac{\sin \alpha}{\alpha}\right) = 1$$

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin \beta} = N$$

The position of maxima –

$$\frac{\cancel{\pi}}{\lambda}(e+b)\sin\theta = \pm n\cancel{\pi}$$

$$\Rightarrow \boxed{(e+b)\sin\theta = \pm n\lambda} \quad \text{— (5)}$$

It is called the grating eqⁿ.

(II) Position of minima –

For the minima condition.

$$\sin N\beta = 0$$

$$N\beta = \pm m\pi$$

$$\beta = \pm \frac{m}{N}\pi \quad \{m = 0, 1, 2, \dots N-1\}$$

As we know the value of β –

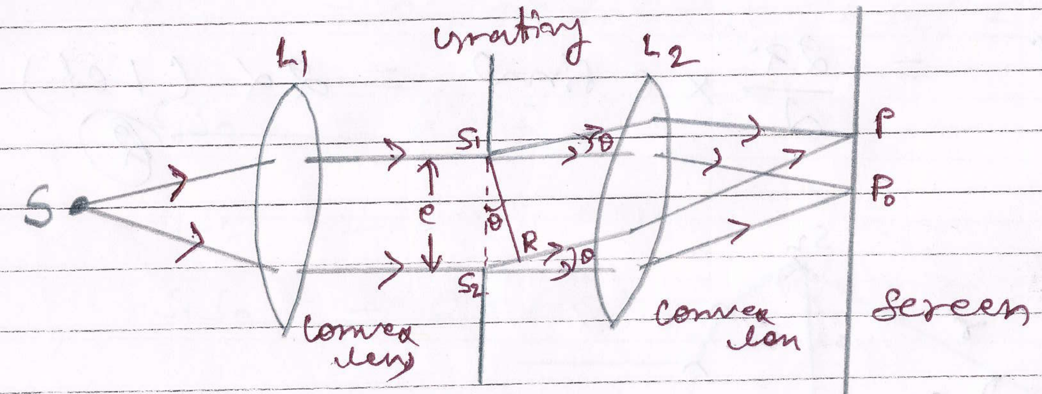
$$\beta = \frac{\cancel{\pi}}{\lambda}(e+b)\sin\theta = \pm \frac{m}{N}\cancel{\pi}$$

$$\Rightarrow \boxed{(e+b)\sin\theta = \pm \frac{m}{N}\lambda} \quad \text{— (6)}$$

It is called the position of minima.

(C) Fraunhofer diffraction at single slit ⇒

When the mono-chromatic light of wavelength λ is incident on the single slit which having the width of 'e'. The light ray will be diffracted at an angle θ, from the Huygen's principal, the every point of wavefront act as a secondary source and emitt the secondary wavelets. These secondary wavelets diffract and produce the diffraction pattern in form of maxima and minima intensity.



The path difference is increased from the point S_1 to extreme point S_2. so the path difference is →

from ΔS_1S_2R ~~or~~ ΔS_1RS_2 →

$$\sin\theta = \frac{RS_2}{S_2S_1}$$

$$RS_2 = S_1S_2 \sin\theta \qquad \left\{ \begin{array}{c} \text{Here} \\ S_1S_2 = e \end{array} \right\}$$

$$RS_2 = e \sin\theta$$

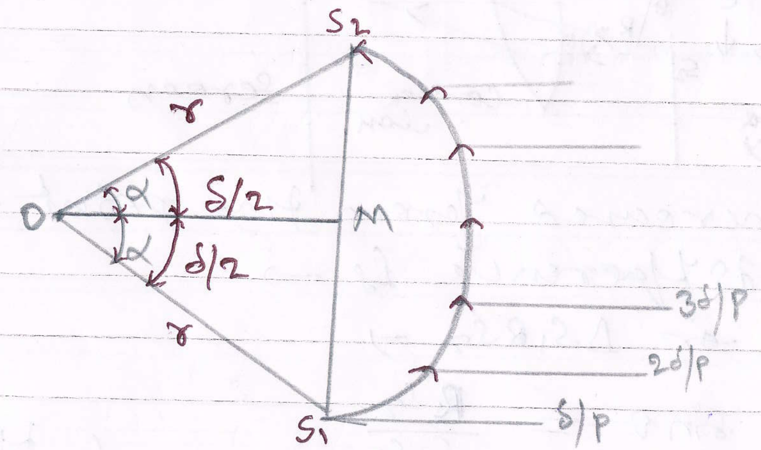
or $$\boxed{\Delta = e \sin\theta} \quad \text{——— (1)}$$

<u>Polygon method →</u>

According to polygon method, the slit S_1S_2 is divided into p small elements, the maximum phase difference b/w the S_1 and S_2 is –

$$\delta = \frac{2\pi}{\lambda} \times \Delta$$

$$\delta = \frac{2\pi}{\lambda} \times e \sin\theta = 2\alpha \text{ (Let)} \quad \text{——— (2)}$$



The amplitude of each element is a, so from the fig –

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

$$2\alpha = \frac{pa}{r}$$

$$\Rightarrow \boxed{r = \frac{pa}{2\alpha}} \longrightarrow (2)$$

From ΔOMS_1,

$$\sin\alpha = \frac{MS_1}{OS_1} \qquad \left\{ \begin{array}{l} \text{Here} \\ OS_1 = OS_2 = r \\ S_1S_2 = 2MS_1 \end{array} \right.$$

$$MS_1 = OS_1 \sin\alpha$$

So,

$$S_1S_2 = 2MS_1$$

$$S_1S_2 = 2\,OS_1 \sin\alpha$$

$$S_1S_2 = 2r\sin\alpha \qquad \left\{ r = \frac{pa}{2\alpha} \right.$$

$$S_1S_2 = 2\frac{pa}{2\alpha}\sin\alpha$$

or $$S_1S_2 = pa\left(\frac{\sin\alpha}{\alpha}\right)$$

As we know the intensity is proportional to the square of amplitude →

$$I \propto (S_1S_2)^2$$

$$I = p^2 a^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

or $$\boxed{I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2} \quad \text{--- (4)}$$

(I) Intensity of secondary maxima → To calculate the secondary maxima, the differentiate w.r. to α and equate it to zero of eqn (4) we have →

$$\frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha}\left(I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2\right) = 0$$

$$2 I_0 \left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\alpha\cos\alpha - 1.\sin\alpha}{\alpha^2}\right] = 0$$

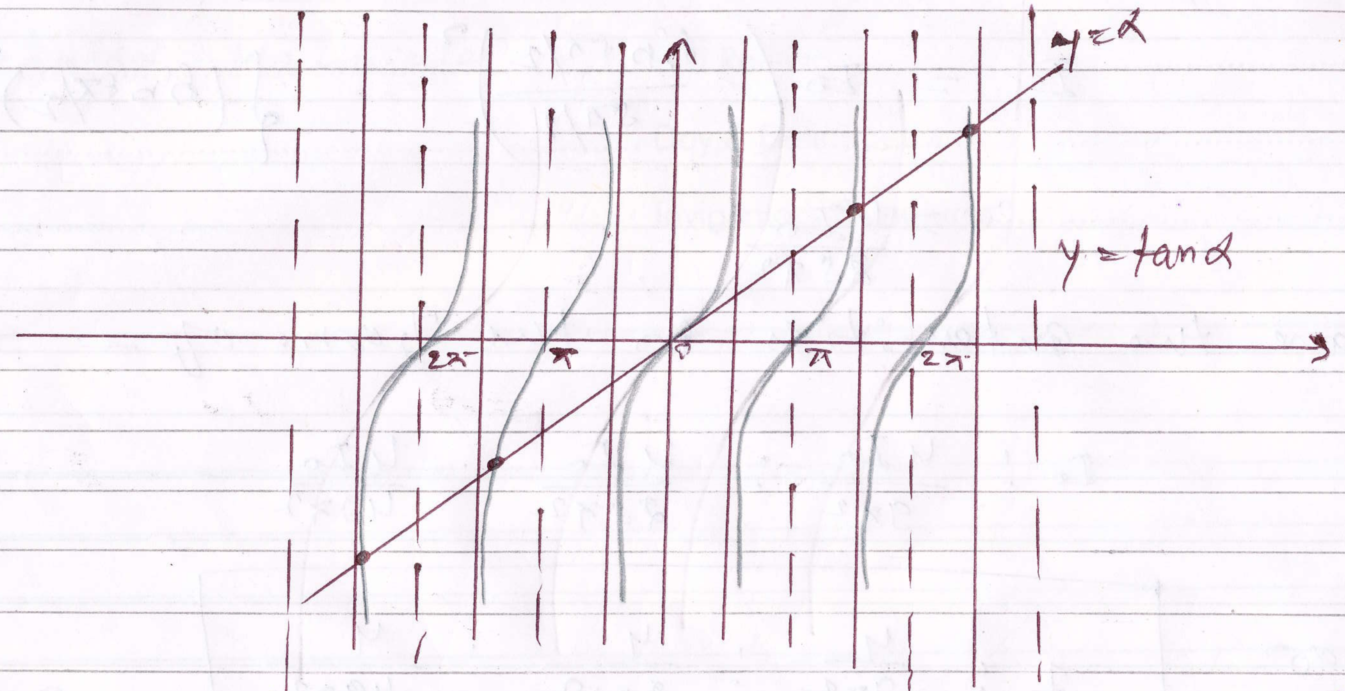
Here (i) $\alpha \neq 0$ the position of central maxima

(ii) $\frac{\sin\alpha}{\alpha} \neq 0$ the position of minima ie,

$$\alpha \cos\alpha - \sin\alpha = 0$$

$$\Rightarrow \alpha = \tan\alpha$$

It can be solved by plotting graph of $y = \alpha$, $y = \tan\alpha$ on single graph →



From the graph, the secondary maxima will be at –

$$\alpha = 3\pi/2,\ 5\pi/2 \ \text{——}\ (2n+1)\pi/2$$

We have the intensity is –

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$= I_0\left(\frac{\sin 3\pi/2}{3\pi/2}\right)^2 = \frac{4I_0}{9\pi^2} \quad \left\{\alpha = 3\pi/2\right\}$$

If $\alpha = 5\pi/2$

$$I = I_0\left(\frac{\sin 5\pi/2}{5\pi/2}\right)^2 \quad \left\{\left(\sin 5\pi/2\right)^2 = 1\right\}$$

$$= \frac{4I_0}{25\pi^2}$$

So, we have the intensities in the form of —

$$I_0 : \frac{4I_0}{9\pi^2} : \frac{4I_0}{25\pi^2} : \frac{4I_0}{49\pi^2} \text{ ———}$$

or $$\boxed{1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2}}$$

Proved